

NUMBER SYSTEMS

- 1. Natural Numbers or Counting Numbers:** These are numbers used for counting things. They include: 1, 2, 3, 4, 5, etc.
- 2. Whole Numbers:** These are natural numbers together with zero. They include: 0, 1, 2, 3, 4, etc.
- 3. Integers:** These are whole numbers together with negatives. They include: -2, -1, 0, 1, 2, etc.
- 4. Square number or perfect square** is the number got after multiplying an integer by itself. They include: 1, 4, 9, 16, 25, etc
- 5. Cube number** is the number got after multiplying an integer by itself three times. They include: 1, 8, 27, 64, etc
- 6. Even Number:** This is a number that is exactly divisible by 2. They include: 2, 4, 6, 8, etc
- 7. Odd Number:** This is a number that is not even. They include: 1, 3, 5, 7, 9, 11, etc
- 8. Consecutive numbers:** These are numbers which follow each other in a given counting order
- 9. Triangular numbers:** These are the consecutive sums of natural numbers. They include: 1, 3, 6, 10, 15, 21, etc and they are obtained as follows:
1, 1 + 2, 1 + 2 + 3, Thus the n^{th} triangular number is given by $\frac{1}{2}n(n + 1)$

EXAMPLES:

- 1.** The sum of two consecutive natural numbers is **101**. Find the numbers
- 2.** The sum of three consecutive odd numbers is **63**. Find the numbers
- 3.** The sum of three consecutive multiples of **5** is **90**. Find these multiples

Soln:

If the multiples are x , $(x + 5)$ and $(x + 10)$,

4. Two consecutive odd numbers are such that six times the smaller exceeds twice the larger by **16**. Find the numbers
5. Find the positive difference between the **30th** and **40th** triangular number
6. The **n^{th}** term of a sequence is given by 2^n
 - (i) Write down the first, third and fifth terms
 - (ii) If the **n^{th}** term is **512**, find the value of **n**

EER:

1. The sum of four consecutive odd numbers is **96**. Find the numbers
2. The sides of a triangle are consecutive odd numbers. Find the longest side, if its perimeter is **45cm**.
3. Find two consecutive even numbers such that seven times the smaller is four less than six times the greater.
5. Find the positive difference between the **20th** and **50th** triangular number
6. The **n^{th}** term of a sequence is given by 3^n
 - (i) Write down the first, third and fifth terms
 - (ii) If the **n^{th}** term is **243**, find the value of **n**

SEQUENCES (NUMBER PATTERNS)

This is an arrangement of numbers in a particular order

EXAMPLES:

1. Find the next numbers in the following sequences:
 - (i) **3, 7, 11, 15, 19, —, —**
 - (ii) **1, 4, 9, 16, —, —**

(iii) 2, 3, 1, 4, 0, —, —

(iv) 1, 8, 27, 64, —, —

(v) 81, -27, 9, -3, —, —

(vi) 1, 4, 20, 120, —, —

(vii) 1, 3, 6, 10, —, —

(viii) $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \text{---}, \text{---}$

(ix) $\frac{1}{2}, \frac{5}{6}, 1\frac{1}{6}, 1\frac{1}{2}, \text{---}, \text{---}$

FACTORS AND MULTIPLES

Summary:

1. *Factors of a number are all numbers that divide exactly into it. 48 is divisible by 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48. Therefore the factors of 48 include: 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.*

2. *Prime Number: This is a number with only two factors one and itself. They include: 2, 3, 5, 7, etc*

3. *Prime factor: This is a factor which is a prime number.*

4. *Prime factorization expresses a number as a product of only its prime factors*

5. *Composite Number: This is a number with more than two factors. Composite numbers can be expressed as a product of their prime factors.*

5. *Rectangle Numbers: These are non prime numbers except 1. They include: 4, 6, 8, 9, 10, etc*

6. *A multiple of a number is that number multiplied by an integer. The multiples of 5 include 5, 10, 15, 20, 25, 30, etc*

7. *Lowest common multiple (LCM) is the lowest multiple two or more*

numbers have in common

8. Highest common factor (HCF) is the highest number that divides exactly into two or more numbers. The highest common factor is also called the greatest common factor **or** the greatest common divisor.

EXAMPLES:

1. List all the factors of each of the following numbers:

(i) 12 (ii) 32 (iii) 60 (iv) 80 (v) 90

2. Express each of the following numbers as a product of its prime factors:

(i) 108 (ii) 441 (iii) 840 (iv) 132 (v) 1440 (vi) 1728

FINDING THE SQUARE ROOTS AND CUBE ROOTS BY PRIME FACTOR METHOD

Summary:

(i) The square root of a number x is denoted by $\sqrt{x} = x^{\frac{1}{2}}$

(ii) The cube root of a number x is denoted by $\sqrt[3]{x} = x^{\frac{1}{3}}$

(iii) When the number is expressed in power form, to find the square root **or** cube root multiply the power by $\frac{1}{2}$ **or** $\frac{1}{3}$ respectively.

EXAMPLES:

1. Express **1225** as a product of its prime factors, hence find the square root of **1225**.

2. Express **1089** as a product of its prime factors, hence find the square root of **1089**.

3. Use the prime factor method to find the square root of:

(i) 2025 (ii) 9801 (iii) $\frac{2178}{2450}$ (iv) 6.25

4. Express **3375** as a product of its prime factors, hence find the cube root of **3375**.

5. Express **1728** as a product of its prime factors, hence find the cube root of **1728**.

6. Use the prime factor method to find the cube root of:

(i) 9261 (ii) 74088 (iii) 0.091125

EER:

1. Find the square root of: (i) $3^6 \times 5^4$ (ii) $2^6 \times 3^2 \times 5^4$

2. Use the prime factor method to find the square root of:

(i) 1225 (ii) 3025 (iii) 0.0625

3. Find the cube root of: (i) $3^6 \times 5^9$ (ii) $2^9 \times 3^6 \times 5^3$

4. Use the prime factor method to find the cube root of:

(i) 10648 (ii) 42875 (iii) 0.035937

5. The sum of three consecutive multiples of **5** is **90**. Find these multiples

HCF AND LCM

EXAMPLES:

1. Find the **HCF** of **210** and **360**

2. Find the **HCF** of **108**, **180** and **216**

3. Find the **HCF** of **210**, **294** and **336**

4. Find the **LCM** of **36** and **60**

5. Find the **LCM** of **27** and **45**
6. Find the **LCM** of **24**, **30** and **40**
7. Find the **HCF** and **LCM** of **36** and **48**
8. Find the **HCF** and **LCM** of **90**, **126** and **270**
9. The **LCM** of two numbers is **180** and their **HCF** is **12**. If one of the numbers is **36**, find the other number.
10. The **LCM** of three numbers is **240** and their **HCF** is **4**. If two of the numbers are **16** and **24**, find the third number.
11. The **LCM** of three numbers is **18900** and their **HCF** is **45**. If two of the numbers are **675** and **540**, find the third number.

WORD PROBLEMS INVOLVING HCF AND LCM

EXAMPLES:

1. Two wires with lengths of **42cm** and **60cm** are to be cut into pieces of equal length without remainder. Find the longest possible length of the pieces.

HINT: The longest length = **HCF**

2. A rectangular plot of land measures **600cm** by **240cm**. The plot is to be divided into square plots of land of equal size. Find the:

- (i) length of the largest possible square plot
- (ii) number of plots that can be got

HINT:

(i) The largest length = HCF

(ii) Number of plots = $\frac{\text{land area}}{\text{square plot area}}$

*3. In a school choir there are **48** girls and **64** boys to be arranged in equal rows each with only girls **or** boys. Find the largest number of students that could be in each row*

*4. Three bells ring at intervals of **24** minutes, **30** minutes and **40** minutes. If they ring together at **8:00 am**, at what time will they ring together again?*

***HINT:** Required time = LCM*

*5. On a circular track, racing car **P** completes the track in **28** seconds, while racing car **Q** completes it in **24** seconds. If they both start at the same time, after how many seconds will they be side by side again?*

***HINT:** Required time = LCM*

*6. Two neon lights are turned on at the same time. One blinks every **4** seconds*

*and the other blinks every **10** seconds. In **60** seconds, how many times will they*

blink at the same time?

*7. Find the smallest number which leaves a remainder of **3** when divided by **4**, **8** and **10**.*

Soln:

***HINT:** Required number = LCM + 3*

*8. Find the smallest number which leaves a remainder of **5** when divided by*

15 and 18.

Soln:

HINT: Required number = LCM + 5

EER:

- 1. Find the HCF and LCM of 108 and 180**
- 2. List down the first 7 multiples of 10, 12 and 15. Hence state their LCM**
- 3. List down the factors of 60 and 90. Hence state their HCF**
- 4. Find the HCF and LCM of 90, 135 and 225**
- 5. Find the least number which is divisible by 40, 50 and 80.**
- 6. Find the smallest number which leaves a remainder of 5 when divided by 12, 15 and 18.**
- 7. The LCM of two numbers is 1260 and their HCF is 18. If one of the numbers is 180, find the other number.**
- 8. The LCM of three numbers is 7920 and their HCF is 12. If two of the numbers are 48 and 264, find the third number.**
- 9. The LCM of three numbers is 900 and their HCF is 6. If two of the numbers are 36 and 60, find the third number.**
- 10. Three bells ring at intervals of 5 minutes, 8 minutes and 10 minutes. If they ring together at a certain moment, find the time that will elapse before they do so again.**
- 11. Three bells ring at intervals of 20 minutes, 24 minutes and 30 minutes. If they ring together at 1:00 pm, at what time will they ring together again?**
- 7. Two wires with lengths of 448cm and 616cm are to be cut into pieces of**

all the same length without remainder. Find the greatest possible length of the pieces.

*8. On a circular track, racing car **P** completes the track in **28** seconds, while racing car **Q** completes it in **24** seconds. If they both start at the same time, after how many seconds will they be side by side again?*

*9. A room which measures **540cm** by **420cm** is to be covered with square tiles of equal size. Find the:*

(i) length of the largest possible square tile that can be used

(ii) number of tiles needed

*10. Find the shortest wire length that can exactly be cut into pieces of equal lengths of **9cm** or **12cm** or **15cm**.*

*11. In a certain school, it is possible to divide the students into groups of equal numbers of **24**, **30** or **32** and have no student left over. Find the least number of students in the school that makes this possible.*

*12. Two neon lights are turned on at the same time. One blinks every **4** seconds*

*and the other blinks every **6** seconds. In **60** seconds, how many times will they*

blink at the same time?

*13. Kiara baked **30** oatmeal cookies and **48** chocolate chip cookies to package in*

plastic containers for her friends at school. She wants to divide the cookies into

*identical containers so that each container has the **same number of each***

kind of

*cookie. If she wants each container to have the **greatest number of cookies***

***possible**, how many plastic containers does she need?*

14. Bridget has swimming lessons every fifth day and diving lessons every third

day. If she had a swimming lesson and a diving lesson on May 5th, when will be

*the next date on which she has **both swimming and diving lessons**?*

*15. Boxes that are 12 inches tall are being stacked next to boxes that are 18 inches tall. What is the **shortest height** at which the two stacks will be the **same height**?*

*16. Tom baked 30 oatmeal cookies and 48 chocolate chip cookies to package in plastic containers for her friends at school. She wants to divide the cookies into identical containers so that each container has the **same number of each kind of cookie**. If she wants each container to have the **greatest number of cookies possible**, how many plastic containers does she need?*